Cox-PH Model

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Introduction

- The Cox PH model is written in terms of the hazard model formula.
- This model gives an expression for the hazard function at time t for an individual with a given specification of a set of explanatory variables denoted by X
- The model is given as

$$h(t, X) = h_0(t)e^{\sum_{i=1}^{p} \beta_i X_i}$$

Cont'd

- The term $h_0(t)$ is the baseline hazard function.
- The explanatory variables are said to time-independent.
- An important feature of this formula,which concerns the proportional hazards (PH) assumption, is that the baseline hazard is a function of t, but does not involve the X's.
- The Cox model formula has the property that if all the X's are equal to zero, the formula reduces to the baseline hazard function. That is, the exponential part of the formula becomes e to the zero, which is 1.

Cont'd

- A time-independent variable is defined to be any variable whose value for a given individual does not change over time. Example is gender.
- The baseline hazard $h_0(t)$ is an unspecified function, this property makes the Cox model a **semiparamtric** model.

Relating Survival and Hazard Functions

- The cumulative hazard is

$$\begin{aligned}
\Lambda(t|X) &= \int_0^t \lambda(t|X)dt \\
&= \int_0^t \lambda_0(t)\exp(X'\beta)dt \\
&= \{\int_0^t \lambda_0(t)dt\}\exp(X'\beta) \\
&= \Lambda_0(t)\exp(X'\beta)
\end{aligned} \tag{1}$$

- Here $\Lambda_0(t)$ is called the baseline cumulative hazard function.

Survival Function

- Let's derive the survival function in this scenario

$$S(t|X) = \exp\{-\Lambda(t|X)\} = \exp\{-\Lambda_0(t)\exp(X'\beta)\}$$

- The density function is given by:

$$\begin{aligned} f(t|X) &= -\frac{d}{dt}S(t|X) \\ &= -\frac{d}{dt}\exp\{-\Lambda_0(t)\exp(X'\beta)\} \\ &= \exp\{-\Lambda_0(t)\exp(X',\beta)\}\lambda_0(t)\exp(X'\beta) \\ &= S(t|x)\lambda(t|X) \end{aligned} \quad (2)$$

Cox PH Model Estimation

- For the observed data (V_i, Δ_i, X_i) $i = 1, ..., n$ the likelihood for the Cox PH model is

$$
\begin{aligned}
L(\beta) &= \prod_{i=1}^{n} f^{\Delta_i}(V_i|X_i)\{S(V_i|X_i)\}^{1-\Delta_i} \\
&= \prod_{i=1}^{n} \{\lambda(V_i|X_i)S(V_i|X_i)\}^{\Delta_i}\{S(V_i|X_i)\}^{1-\Delta_i} \qquad (3) \\
&= \prod_{i=1}^{n} \{\lambda_0(V_i \exp(X_i'\beta)\}^{\Delta_i} \exp\{-\Lambda_0(V_i \exp(X_i')\}
\end{aligned}
$$

Thank You!